SECTION 14.3: MOTION IN SPACE

POSITION, VELOCITY, SPEED, ACCELERATION:

- $\vec{r}(t)$, $t \in [a, b]$ is the **position** function; $\vec{r}(t)$ tells where you are at time t.
- $\vec{v}(t) = \vec{r}\,'(t)$ is the **velocity** function; $\vec{v}(t)$ tells you how fast and in which direction you're moving.
- $\|\vec{v}(t)\|$ is the **speed**; $\|\vec{v}(t)\|$ tells you how fast you're going.

  NOTE: Think of $\|v(\vec{t})\|$ as the 'speedometer' function.
- $\hat{T}(t) = \dfrac{\vec{v}(t)}{\|\vec{v}(t)\|}$ is the **principal unit tangent vector**; $\hat{T}(t)$ tells you the direction you're going.

  NOTE: For $\hat{T}(t)$ to exist, $\|v(\vec{t})\| \neq 0$. In this case, $\vec{v}(t) = \|\vec{v}(t)\|\,\hat{T}(t)$.
- $\vec{a}(t) = \vec{r}\,''(t) = \vec{v}\,'(t)$ is the **acceleration function**.

  NOTE: For constant mass, $\vec{F}(t) = m\vec{a}(t)$, so $\vec{a}(t)$ is parallel to the force which generates the path $\vec{r}(t)$.

EXAMPLE 1: UNIFORM (CONSTANT SPEED) LINEAR MOTION:

Suppose an object has a constant velocity $\vec{v}(t) = \langle a, b, c \rangle$ and passes through the point $P\,(x_0, y_0, z_0)$.

Derive a vector-valued function which describes the trajectory of the object.

Ans: $\vec{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$

EXAMPLE 2: NON-UNIFORM LINEAR MOTION: Let $\vec{r}(t) = \langle 1 - \sin(t), 2 + 4\sin(t), -3\sin(t) \rangle$.

1. Show that $\vec{r}$ traces out a linear trajectory.

   Ans: let $u = \sin(t)$ then $\vec{r}(u) = \langle 1 - u, 2 + 4u, -3u \rangle$

2. Show $\|\vec{v}(t)\|$ is not constant.

   Ans: $\|\vec{v}(t)\| = \| \langle -\cos(t), 4\cos(t), -3\cos(t) \rangle \| = \ldots = \sqrt{26}\,|\cos(t)|$

EXAMPLE 3: CIRCULAR MOTION:

Suppose $R > 0$ and $\omega > 0$.

1. Verify $\vec{r}(t) = \langle R\cos(\omega t), R\sin(\omega t)\rangle$ traces out counter-clockwise motion on the circle $x^2 + y^2 = R^2$.

2. Find and simplify expressions for the velocity, $\vec{v}(t)$ and speed, $\|\vec{v}(t)\|$.

Ans: $\vec{v}(t) = \langle -R\omega\sin(\omega t), R\omega\cos(\omega t)\rangle$, $\|\vec{v}(t)\| = R\omega$

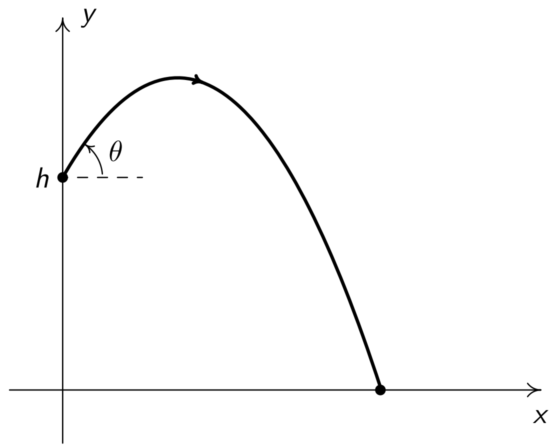
3. Verify $\vec{r}(t) \perp \vec{v}(t)$ and $\vec{a}(t) = -\omega^2\,\vec{r}(t)$.

Ans: $\vec{a}(t) = \vec{v}\,'(t) = \left\langle -R\omega^2\cos(\omega t), -R\omega^2\sin(\omega t)\right\rangle = -\omega^2\,\langle R\cos(\omega t), R\sin(\omega t)\rangle$

EXAMPLE 4: PROJECTILE MOTION:

Suppose an object is launched into the air with initial speed v_0 at an angle θ from the horizontal from a height h off the ground. If we let g denote the acceleration due to gravity and ignore air resistance, we have:

$$\vec{a}(t) = \langle 0, -g \rangle\,, \quad \vec{v}(0) = v_0 \langle \cos(\theta), \sin(\theta) \rangle\,, \quad \vec{r}(0) = \langle 0, h \rangle$$



Use this information to derive a formula for $\vec{r}(t)$.

EXAMPLE 5: JT competes in Highland Games Competitions across the country. In one event, the 'hammer throw', he throws a 56 pound weight for distance. If the weight is released 6 feet above the ground at an angle of 42° with respect to the horizontal with an initial speed of 33 feet per second. (Here, use $g = 32\frac{\text{ft.}}{s^2}$.)

1. Find the v.v.f. which models the trajectory of the hammer. As usual, ignore all forces except gravity.

Ans: $\vec{r}(t) = \left\langle 33\cos(42^\circ)t, -16t^2 + 33\sin(42^\circ)t + 6 \right\rangle$

2. How long is the weight in the air? Round your answer to four decimal places.

Ans: Solving $y = -16t^2 + 33\sin(42^\circ)t + 6 = 0$ for $t > 0$ gives $t \approx 1.6126$ seconds.

3. How far away from JT does the weight land? Round your answer to the nearest inch.

Ans: $x = 33\cos(42^\circ)(1.6126) \approx 39\,\text{ft}\,6\,\text{in}$

HOMEWORK: Section 14.3: 9 - 65 every other odd. 27*, 28* - 'The Scrambler.'